ENVIRONMENTAL NOISE CONTROL
The 2005 Congress and Exposition on Noise Control Engineering
07-10 August 2005 - Rio de Janeiro - Brazil

The Distribution of Absorption Materials in a Rectangular Room

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Abstract Students in Architecture are taught Sabine's formula for the reverberation time (*RT*) and common theory for the sound pressure level (*SPL*), but actually, these equations are for a cubic space with a diffuse sound field and absorption materials distributed homogeneously through the room. The influence of room shape and uneven absorption distribution on *RT* has been investigated for many decades. The consequences for *SPL* have been dealt with much less. A simple mirror sources model is used to derive general rules for a rectangular enclosure. The model predicts *RT* and *SPL* to increase in almost any case. *RT* is mainly influenced by the longest room dimension, while *SPL* is decreased when absorption is perpendicular to the shortest dimension. It explains why ceiling absorption is effective. Some simple adaptations can be made to the common theory to estimate the effect.

1. INTRODUCTION

The common acoustical theory on reverberation in enclosures is based on a linear decay of the sound pressure level when a sound source is switched of. However, deviations are found, both in measurements and in computer based calculation methods, when enclosures are non-cubic or when absorption materials are distributed inhomogeneously in the enclosure. These deviations not only lead to differences in the reverberation time, but the actual sound pressure levels for a constant sound source vary as well.

In the present research at the Faculty of Architecture of our University, the emphasis is on developing acoustical guidelines for enclosures like offices, classrooms, sports facilities, and rooms in institutions for mentally challenged people or people with a hearing loss. In these cases the reverberation time gives a good indication about the acoustical quality of the enclosure, but actually the sound pressure level is much more important. It is the aim of the present article to present some results from a simple mirror sources model to illustrate the decrease and increase of sound pressure levels and to derive guidelines for architects about room shape and distribution of absorptive materials.

2. THEORY

In many text books the theory of sound decay in an enclosure is described. The books of Pierce [1] and Kuttruff [2] are examples to understand the basis of the present paper.
In theoretical derivations the sound pressure level *SPL* in a room is calculated as:

$$SPL = 10\log\left(\frac{W\ rc}{p_{ref}^2}\left(\frac{1}{4pr^2} + \frac{4R}{aS}\right)\right) \tag{1}$$

SPL is built up of a direct sound and a diffuse part, given by the first and the second term within the brackets. *W* is the source power; r and c are the density and speed of air, p_{ref} is the reference sound pressure (2.10^{-5} Pa). The distance between source and receiver is given as r; S is the total surface of the room and a is the mean absorption coefficient of all surfaces. R is the reflection coefficient, which is complementary to the absorption coefficient as $R = 1 - a$.
The theoretical derivation is based on the increase or decrease of sound energy in a room when a sound source is switched on or off. If only the second term is considered (so for large source-receiver distances), the theoretical decay can be written as a function of time t:

$$SPL(t) = 10\log\left(\frac{W\ rc}{p_{ref}^2}\left(\frac{4R}{aS}\right)\exp(-b\ t)\right) \tag{2}$$

where two values for the decay variable b are commonly used:

$$b_{sab} = \frac{c\,a}{l_{mfp}} \qquad \text{and:} \qquad b_{eyr} = \frac{-c\ln R}{l_{mfp}} \tag{3a, b}$$

for the Sabine and Eyring reverberation times respectively.
The “mirror sources model” is based on a similar derivation. The total sound power is found at the microphone position when the power contributions are summed over all possible sound sources found by geometrical rules. When all mirror sources (denoted by i) are added, the total sound power E can be written as:

$$E = \frac{W\ rc}{p_{ref}^2}\sum_i \frac{R^{n_i}}{4pr_i^2} \tag{4}$$

where n_i denotes the number of reflections for mirror source i. The direct sound is found for $i = 0$. It may or may not be included, but is left out in the majority of our calculations.
In a cube with the absorption homogeneously distributed along the surfaces, n_i can be given as $n_i = r_i / l_{mfp}$, where l_{mfp} stands for the mean free path, calculated as $l_{mfp} = 4V/S$. Note that for a cube $l_{mfp} = 2/3\ L$, where L is the dimension of the cube.

Now let us assume that the space is rectangular with dimensions $L_x \times L_y \times L_z$., and, for convenience, $L_x \geq L_y \geq L_z$. Furthermore it is assumed that there are only three reflection coefficients R_x, R_y en R_z for the surfaces perpendicular to the *x*, *y* and *z* directions. The contribution to the sound power of one mirror source *i* is now written as:

$$E_i = \frac{W\,\rho c}{p_{ref}^2} \frac{R_x^{n_x} R_y^{n_y} R_z^{n_z}}{r_i^2} \qquad (5)$$

where n_x, n_y en n_z represent the number of reflections in the three directions.

Again an attempt can be made to calculate the decay similar to Eq. (2). However, a solution for the underlying integral has never been found, not even for the perfect cube. Solving the integral numerically is not too difficult, but the summation of Eq. (4) is even simpler. This subject is beyond the contents of the present article, but three different decay values emerge, which are similar to Eyring's value:

$$b_x = \frac{-c \ln R_x}{L_x} \qquad b_y = \frac{-c \ln R_y}{L_y} \qquad b_z = \frac{-c \ln R_z}{L_z} \qquad (6a, b, c)$$

They are given here because there is one interesting special case, when $b_x = b_y = b_z$. Obviously this is found for a cube with all absorption homogeneously along the surfaces, but it is found also in non-cubic spaces if the dimensions are compensated by the absorption.

3. NUMERICAL RESULTS FROM THE MIRROR SOURCES MODEL

3.1 Numerical results for a rectangle

Equations (4) and (5) can be evaluated in a numerical process to derive decay curves if a simple conversion is made whit $r_i = c\,t_i$.

The first curves from the model are for a cubic space of 10×10×10 m^3. All surfaces have the same absorption coefficient. These chosen dimensions are not very useful from the architectural viewpoint. However, results can be scaled up and down with a linear scale factor as long as *SPL* and the reverberation time are scaled accordingly.

The left part of figure 1 shows the results for three values of the absorption coefficients: 10%, 30% and 90%. In the numerical process the influence of the direct sound is left out as was also done in the theoretical Eqs. (2) and (4). The curves (full lines) are compared with a curve from Sabine's theory, according to Eqs. (2) and (3a). The lower dotted line is for Eyring's decay value (Eqs. 2 and 3b). The horizontal parts at the start of the curves are because it takes some time for the first reflections to arrive at the receiver position. In this case this is found as $t_{mfp} = l_{mfp} / c = 19.5$ ms.

The decay of the mirror sources model is not linear, although deviations are small. This can be seen from the 30% case of figure 1, but a closer look at the 10% and 90% cases show the same effects. We found no physical or mathematical proof, but our hypothesis is that curves are always concave.

The first part of the decay curve from the mirror sources model agrees better with Eyring's value than with Sabine's. However, when a mean value is calculated over the later part of the decay curve, Sabine's decay value is a better estimate. This is best demonstrated by the 30% curve, but is found in many other cases as well. This is probably also the reason why in practice Sabine's reverberation time is used more often than Eyring's value.

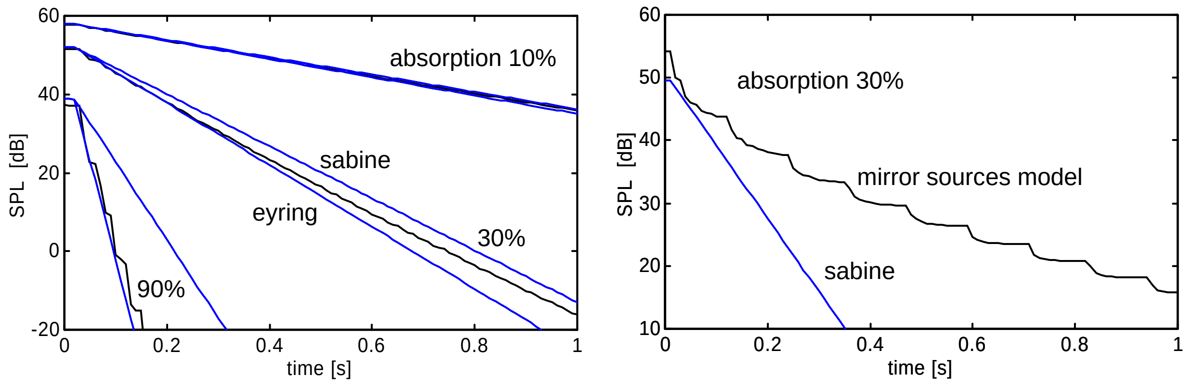

Figure 1: Decay curves for a cubic space of 10´10´10 m^3 (left) and for a space of 40´10´2.5 m^3. Absorption is homogeneously distributed along all surfaces.

In the right part of figure 1 the calculations have been repeated for a room with dimensions 40×10×2.5 m^3 (so with equal volume). All surfaces have an absorption coefficient of 30%, which means that the total absorbing surface increases from 180 m^2 to 315 m^2. According to Eq. (1) *SPL* should decrease from 51.9 to 49.5 dB and T_{sab} from 0.90 to 0.51 s. However, this is not the case as both *SPL* and T_{sab} increase considerably. The increase in sound pressure level is because the first mirror source is much closer to the receiver than in the cubic space.

The reverberation time is calculated from the slope of the decay curve. Since straight curves are not found, the slope is found by statistical curve fitting. The early decay time (*EDT*) is fitted along the interval between the top 0 dB-level at t = 0 and the -10 dB level. The official reverberation time should be fitted between -5 and -35 dB, although in practice a shorter interval is often used if the signal to noise ratio is insufficient (-5 to -25 dB). Table 1 shows the values calculated from both curves. They are much larger than the values from Sabine's and Eyring's equations, except for *EDT* from the mirror sources model.

Table1: Reverberation times for a rectangular room (40´10´2.5 m^3), with 30% absorption coefficient. Sabine and Eyring values are calculated with Eqs. (3a, b). Mirror sources values are from curve fitting along the curve of figure 1 (left), for four decay intervals.

Sabine	Eyring	mirror sources model			
		0 ⇒ -10	-5 ⇒ -15	-5 ⇒ -25	-5 ⇒ -35
0.51	0.43	0.53	1.07	1.44	1.84

The increase of the reverberation time has been a subject of quite some investigations. There is even a European standard method, laid down in NEN 12354-6 [3], based mainly on Nilsson's work [4]. However, surprisingly little attention has been paid to the increase in sound pressure levels. Our emphasis is on this subject since it is at least equally important for the abatement of noise.

3.2 A quasi-homogeneous case and ceiling absorption

As mentioned earlier, there is one special case when $b_x = b_y = b_z$ (given in Eqs. 6). This is illustrated in the left hand side of figure 2. As a basis the curve "cube" is given, which is equal to the central curve in figure 1. For the curve "quasi-homogeneous" the input in the y-direction is taken the same, so L_y = 10 and a_y = 0.30. But now L_x = 40 and L_y = 2.5, and the absorption coefficients are found as a_x = 0.76 and a_z = 0.085.
In the cubic space the value of T_{25} was found as 0.81 s. This value is between Sabine's and Eyring's values (0.90 and 0.78 respectively). As can be seen from the left hand side of figure 2, the value of T25 remains more or less the same (T_{25} = 0.79 to be exact). However, Sabine's and Eyring's values increase, because the amount of absorption surface is decreased from 180 to 166 m2, to 0.98 and 0.90 s.
It is a good means to choose $b_x = b_y = b_z$ in order to minimize the reverberation time, but unfortunately it is not a good means to decrease *SPL*. The value of *SPL* is in the 40×10×2.5 case 5.5 dB higher than in the 10×10×10 cube (56.9 versus 51.4 dB). The reason is that the first mirror sources are close to the receiver.

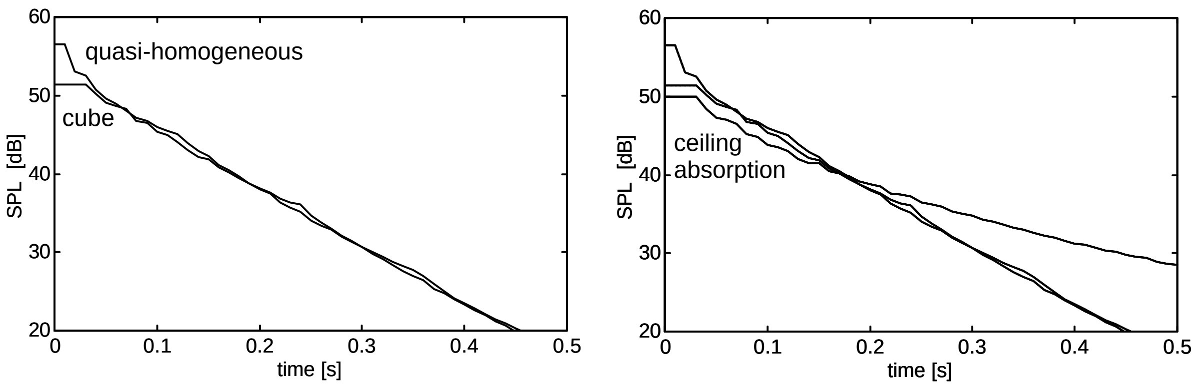

Figure 2: Decay curves when a cubic space with homogeneous absorption is compared with a rectangle with non-homogeneous absorption. See text for explanation.

In the right hand side of figure 2, the two curves of the left figure are repeated as dotted curves. One curve has been added where a_x and a_z have been interchanged (a_x = 0.085 and a_z = 0.76.), so the classical case of ceiling absorption is introduced.
The three values of *b* are very different, so a strong concave curve is found. The total absorbing surface is very high (672 m2) and hence Sabine's reverberation time is as low as 0.24 s and *SPL* is only 43.3 dB. The results from the mirror sources model do not agree at all: T_{25} = 2.79 and *SPL* from Eq. (2) = 49.9 dB.

In figure 3 *SPL*'s from the numerical model are compared with *SPL*'s from Eq. (2). In the left figure the influence of non-uniform absorbing material is shown in a 10×10×10 m^3 cubic space. In the basis situations all absorption coefficients are 20%. *SPL*-values vary considerably as the total amount of absorbing surface may vary. However, deviations from *SPL* from Eq. (2) appear very small. It is interesting to see that *SPL*'s from the mirror sources model are somewhat lower than *SPL*'s from common theory.
The influence of shape appears much more important as shown in the right side of figure 3. In this case all surfaces have a 20% absorption coefficient. Deviations between the mirror

sources model and common theory are as high as 11.5 dB, although this last value is for a somewhat peculiar space: 80×10×1.25 m3.

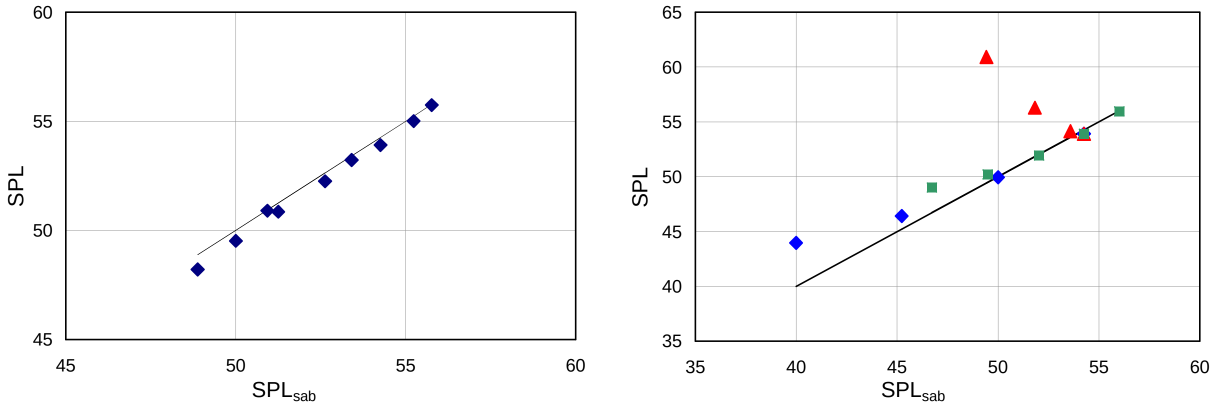

Figure 3: Comparison of the mirror sources model with values from common theory, denoted by SPL_{sab}. The source power level is taken as 70 dB (re 10^{-12} W). In the left figure all spaces are cubic (10´10´10 m^3). The basis is the (54.3, 53.9) value where all surfaces have the same absorption coefficient of 20%. Other dots represent a variety of absorption coefficients with 20 / 20 / 99% as an extreme case.
In the right figure all surfaces have the same absorption coefficient 20%. The basis form is cubic (10´10´10 m3) , resulting in the (54.3, 53.9) value. Triangles represent a constant volume of 1000 m^3, with extreme dimensions 80´10´1.25; squares vary from 5´10´10 to 80´10´10 m^3; diamonds are for square room: from 10´10 to 80´80 with equal heights of 10 m.

3.3 Source-receiver distance

Eq. (1) contains a term for the direct sound with a given distance between the source and the receiver. For the second term in that equation it is implicitly assumed that the source and receiver are in the center of the room. So far, this assumption has also been made for the mirror sources model, but it is easy to drop this assumption by using an extended version of Eq. (4). In figure 4 results are presented for a room with a 40×40 m^2 floor surface. In the left figure a comparison is made with results from common theory. In the right hand figure results are compared for ceiling heights 2.5 and 10 m.

The mirror sources model predicts an increase of 1 to 2 dB (compared with common theory) at 1m from the source. This is found in measurements as well. The differences between common theory and the mirror sources model are quite strong between x = -10 and -5. It depends on the architectural function of the space if the effect should be considered as positive or negative. Around a conference table an increase of *SPL* may be welcomed; for a restaurant it is probably a nuisance. Increasing ceiling height may be another tool to reduce *SPL*, which is shown in figure 3, right.
When x is of the order of +15, the mirror sources model predicts lower values than common theory. This effect was dealt with in more detail in a separate paper [5], where corridor-like shapes and L- and U-shapes were used to decrease *SPL*'s. It is outside the scope of the present paper.

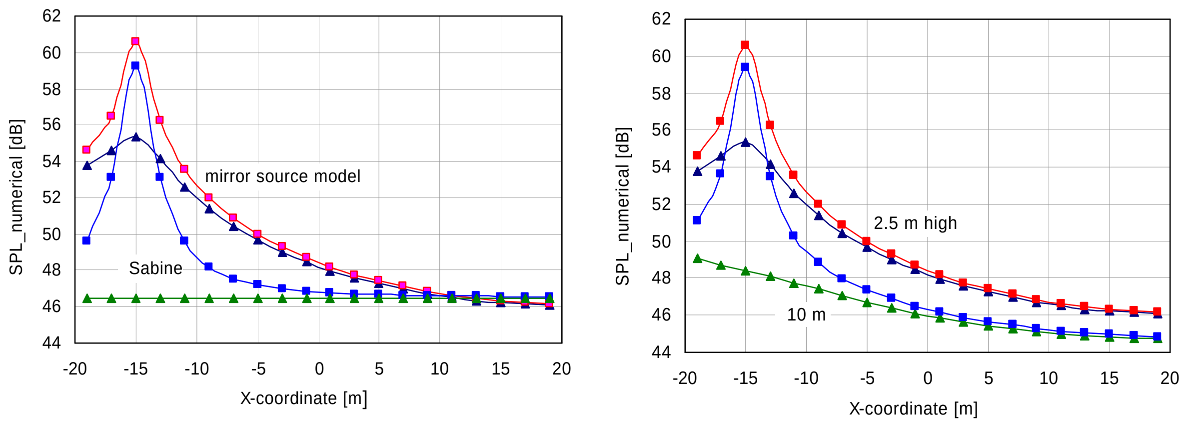

Figure 4: Calculations for a room with x and y coordinates ranging from -20 to +20 m. Source position is at (-15, -15), the microphone is at y = -14, while x-values range from -19 to +19; so the shortest distance is 1 m and since L_W = 70, SPL is predicted by common theory as 59.0 dB. All surfaces have the same absorption coefficient of 20%.
In the left figure comparisons have been made with results from common theory (called "Sabine") for a room with 2.5 m ceiling height. Calculations have been done with and without the direct sound, respectively given by squares and triangles. In the right figure two ceiling heights are compared: 2.5 and 10 m, again with and without the direct sound included.

3.4 An adapted version of common theory

Attempts have been made to find an adapted version of Eq. (2) to calculate the influence of shape. Therefore a figure has been drawn with the results from the mirror sources model (relative to *SPL*'s from common theory) along the horizontal axis. For the variable along the vertical axis different methods have been tried. The final result is given in figure 5.

Since shape appears very important in figures 3, the first variable that comes into mind is the mean free path. So the following substitution was made:

$$\frac{R}{S} \Rightarrow \frac{R}{6 \times \left(1.5\ l_{mfp}\right)^2} \tag{7a}$$

Another substitution is based on the summation of the first order mirror sources:

$$\frac{R}{S} \Rightarrow \frac{1}{18}\left(\frac{R_x}{L_x^2} + \frac{R_y}{L_y^2} + \frac{R_z}{L_z^2}\right) \tag{7b}$$

The best estimation appears between these two cases:

$$\frac{R}{S} \Rightarrow \frac{1}{18}\left(\frac{R_x}{L_x^p} + \frac{R_y}{L_y^p} + \frac{R_z}{L_z^p}\right)\frac{1}{\left(1.5\ l_{mfp}\right)^{2-p}} \tag{8}$$

where *p* is some arbitrary exponent.

The highest correlation factor (R^2 is of the order of 95%) is found when the exponent p is of the order of 0.4 to 0.6. However, the correlation curve itself is 1.5 dB too high. Eq. (8) has been chosen to turn into a cubic space if $L_x = L_y = L_z$. This is found in the figure around the (0, 0)-coordinate, but since cubic spaces are very rare in architectural practice it would be more convenient to subtract an extra 1.5 dB.
However, even if the absolute values from Eq. (8) are not fully correct, the equation gives a good indication how shape and absorption positioning could be used in architectural practice. It will be used in the last chapter.

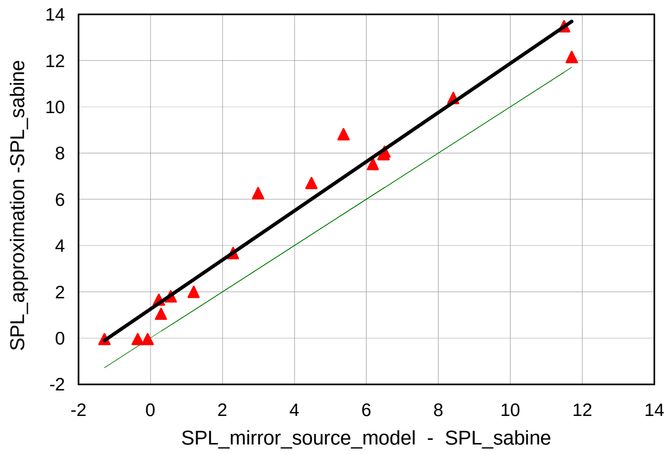

Figure 5: Correlation between the mirror sources model and Eq. (8). Thin line is for an ideal case; the black line is for best fit.

4. BUT, IS THE MIRROR SOURCES MODEL RELIABLE?

Concave decay curves are found very often when measuring rooms. Especially in corridor-like enclosures big deviations from linearity are always found. This is also the reason that measured *EDT*-values can be considerably shorter than reverberation times calculated from 20 or 30 dB intervals. However, The mirror sources model sometimes predicts extreme deviations that we did not found in our own measurements. Because it is always difficult to estimate the absorption coefficients from existing materials in rooms, scale model measurements are performed at present to gain more insight in this subject.
When calculations are done in a ray-tracing model with specular reflecting surfaces results should be the same as those from the mirror sources model. This is confirmed by comparisons made with three models (Raynoise, Epikul, CattAcoustic). However, results from ray-tracing models appear very sensitive to diffusion factors given to surfaces and a mirror sources model does not incorporate diffusion. The effect is illustrated in figure 6. Calculations have been done (in Catt Acoustic) in a rectangular space of 21×12×5.5 m^3. All walls have the same absorption coefficient 0.25.
Results are given in a *SPL*-*RT*-diagram. The dots along the diagonal curve represent the *SPL* and *RT*-values for the diffuse field when different values of a are used. The small square on this curve represents the input value a = 0.25. All calculations have been done for one source position and 20 microphone positions.
If both common theory and ray-tracing models were ideal, all *SPL*-values calculated at different microphone positions, should lie within this rectangle. In figure 6 this appears not the case. Of course, *SPL*-values vary through an enclosure because of the contribution of the di-

rect sound (the first term in Eq. 1). If this were the only reason, all ray-tracing dots would be within the small square if the receiver is in the diffuse field or on the *left* side of this square for small source-receiver distances. Because Eq. (1) is not fully adequate, dots are also found at the right side of the small square.

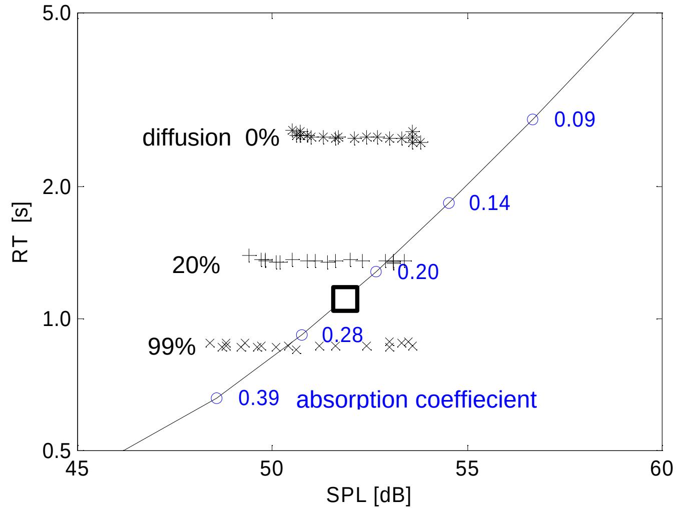

Figure 6: RT and SPL values from a ray-tracing computer model for three different diffusion factors. Room size is 21´12´5.5 m^3; a = 0.25 (small square). There is one source position and 20 microphone positions.

In fact, the biggest deviations are found in the *RT*-values. The reverberation times appear very sensitive to the diffusion factor, which has been a subject of many debates about the possibility to calculate the reverberation time from ray-tracing models. Fortunately, the sensitivity of the *SPL*-values appears much less, probably because the ray tracing model is based on energy calculations and *SPL* is an energy-based variable.

So, an answer to the question whether or not the model sources model can be used for the prediction of *SPL*-values is: it is just as good as a ray-tracing program. Extra scale model measurements must clarify more.

5. CONCLUSIONS FOR ARCHITECTS

Equation (8) has its uncertainties, but even if it is not very accurate to calculate the absolute values of *SPL*, it is useful for architectural practice since comparisons can be made between different room shapes and absorption distributions. Hence the following general conclusions can be drawn:

- An architect has to think about his/her main concern, the reverberation time or the sound pressure level. In a rectangular room the reverberation time depends on the properties along the longest dimension, the sound pressure level on the shortest dimension.
- As always, room shape and absorption distribution depend on architectural function. In many cases high sound pressure levels must be abated, but on some occasions (conference rooms) high levels under a low reflective ceiling may be advantageous.
- If high levels should be avoided (in a restaurant for instance) the first step is to increase ceiling height whenever possible. Eq. (8) depends on the mean free path, which in low rooms depends mainly on the shortest dimension.

- In practice ceiling absorption is often used: it has the largest surface and room cleaning is easy. The first part of Eq. (8) shows why it is effective, but it also shows when absorption on the walls should be added as well.
- The mirror sources model predicts an *in*crease of sound pressure levels, when compared with common theory, when the receiver is close to the source. At bigger distances a *de*crease is found, so the floor plan of a room can be used to reduce noise levels.

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